## **TECHNICAL ARTICLE**



# Using GIS and Fractal Theory to Evaluate Degree of Fault Complexity and Water Yield

Binbin Yang<sup>1</sup> · Junhong Yuan<sup>1</sup> · Lihong Duan<sup>2</sup> · Qin Liu<sup>1</sup>

Received: 3 October 2017 / Accepted: 27 September 2018 / Published online: 5 October 2018 © Springer-Verlag GmbH Germany, part of Springer Nature 2018

#### Abstract

This paper presents a way to evaluate the degree of fault complexity in coal mines using the geographic information system (GIS) and fractal theory. First, the factors that affect the degree of complexity of coal mine faults are determined. Then, an analytic hierarchy process model is used to calculate the factor weighting, and an information entropy model is used to improve the weight information. Based on the local weighted linear combination, a fault complexity index is calculated. The proposed method was validated by a case study at the Chensilou coal mine in Henan Province, China. The result indicates that the method is robust; the degree of fault complexity was quantified and categorized. The fault complexity in the studied area consists of simple and moderate faults and the water yield was found to increase with the complexity index, indicating an increased risk of water inrush.

**Keywords** Fault complexity index · Coal mine · Water inrush · Analytic hierarchy process · Information entropy

## Introduction

In recent years, the frequency of water related incidents in coal mines and the number of related casualties in China have declined (Supplemental Fig. 1; (Network of coal mine safety production in China 2018)). However, water related incidents in mines remain a very serious issue, causing major economic losses and casualties (Wang et al. 2012).

**Electronic supplementary material** The online version of this article (https://doi.org/10.1007/s10230-018-0563-8) contains supplementary material, which is available to authorized users.

☑ Junhong Yuan yjhyjh20@163.com

Binbin Yang yangbinbin@cumt.edu.cn

Lihong Duan dlhcumt@163.com

Qın Lıu liuqin@cumt.edu.cn

- School of Resources and Geosciences, China University of Mining and Technology, 1 University Rd, Xuzhou 221008, Jiangsu, China
- Yongcheng Coal and Electricity Holding Group Co. Ltd, Henan Energy and Chemical Industry Group, Middle Guangming Rd, Yongcheng 476600, Henan, China

About 80% of water inrushes are related to faults (Miao et al. 2004; Xu 2011); hence, it is important to analyze this relationship further.

The degree of coal mine fault complexity is affected by many factors, such as the fractal characteristics of the fault network and strata inclination (State Administration of Coal Mine Safety 2014; Zhao et al. 2011). The characteristics of water inrush from faults, reasons for water inrush, and degree of coal mine fault complexity have been examined by many academics. Both in-situ measurements and numerical modeling have been used to determine the cause of water inrush from faults. For example, Wu et al. (2004) conducted laboratory tests on rock samples from fault zones and then used fast Lagrangian analysis of continua in three dimensions to numerically simulate different scenarios that would cause groundwater inrush from fault zones. Li et al. (2011) used the finite element method to numerically investigate water inrush from fault zones in coal mines. Han et al. (2009) modeled the mechanism of water inrushes from faults in the coal seam floor.

The distribution density of faults has been used to evaluate fault involvement, but this just considers the number of faults (Chen et al. 1999). Faults are a complex natural fracture system that have self-similar properties and can be examined using the fractal theory (Li et al. 2008; Mandelbrot 1979, 1983), like the fracture geometry of rocks (Hirata



1989). The spatial variations of the Anatolian fault zones and their seismicity and fractal dimension were analyzed by Öncel et al. (1996). Sukmono et al. (1996) analyzed the geodynamic processes and variations in the Sumatra active fault system using a fractal approach. Also, research on the relationship between fractal dimensions and spatial distribution of faults has become increasingly common (e.g. Adib et al. 2017; Carpinteri and Paggi 2008, 2009; Pérez-López et al. 2005; Sengupta et al. 2011). The length and dimensions of faults, fault properties, fault interactions, and dynamic mechanisms of faults can be defined using the fractal dimensions of the spatial distribution of fracture systems (Aviles et al. 1987). Wang et al. (2017) examined the fractal characteristics of the spatial distribution of faults associated with water inrush events. Duan and Ran (2013) investigated the relationship between the volume of water flow and the fractal dimension of a fault, but the dip angle of the coal seam and the fault throw were not considered.

In recent years, geographic information systems (GIS) have been widely used in mining related applications (Wu and Zhou 2008; Yang et al. 2017). GIS collect, access, integrate, process, analyze, and display geographic information, so that data can be combined with multi-criteria decision analysis to facilitate decision support systems (Makropoulos and Butler 2006; San Cristóbal 2011). In this paper, the degree of complexity of coal mine faults was evaluated using GIS combined with multi-criteria decision analysis and fractal theory. Moreover, the relationship between water yield and the fault complexity index (FCI) is analyzed.

## **Methods**

This section provides a brief description on the process of establishing a FCI. The FCI model is used to evaluate the degree of fault complexity in coal mines by combining weighted linear combination (WLC), an analytic hierarchy process (AHP), and the fractal theory in a GIS environment.

- Establishment of factors that affect the degree of fault complexity: according to the "Regulations of geological work for coal mines" imposed by the State Administration of Coal Mine Safety (2014), three main factors are provided as indicators that affect the degree of fault complexity in coal mines including: dip angle of the coal seam, fault throw and fractal dimensions of the faults.
- 2. Calculation of the fractal dimensions of faults: the fractal theory and methods have been extensively applied in the field of geotechnical engineering, for geographic analyses (Babanouri et al. 2013; Hossain and Kruhl 2015). The fractal dimensions of the spatial distribution of faults have been combined into an index of the number of faults, fault dimension, and compound modes

(Ben-Zion and Sammis 2003; Guarnieri 2002; Kato and Lei 2011). The fractal dimension can be calculated using box counting, which is defined as:

$$d = \lim_{s \to 0} \frac{\log Ns}{\log(1/s)} \tag{1}$$

where *N*s is the number of grid cells covered by a box with a length of s.

3. Establishment of factor weights based on AHP: the AHP was first proposed by Saaty (1980) in the early 1970s, and is a decision-making method that combines quantitative and qualitative analyses (Uyan 2013). First, a hierarchical structure model is established. Then, a comparison is made between factor elements at each level, and the elements in the comparison matrix are defined as  $A = (a_{ij})_{n \times n}, a_{ij} > 0, a_{ij} = 1/a_{ji}$ :

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$
 (2)

where  $a_{ij}$  denotes the ratio of the *i*th factor to the *j*th factor; the possible evaluated values of  $a_{ij}$  in the pairwise comparison matrix are listed in Supplemental Table 1.  $A_w$  is defined as the normalized matrix, which is transformed using the formula:

$$A_{w} = (a'_{ij})_{n \times n} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}}, i = 1, 2, \dots, n, j = 1, 2, \dots n$$
(3)

where  $w_i$  is defined as the weight of the *i*th factor, and  $W = (w_i)_{1 \le n}$  is calculated as follows:

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} \frac{1}{n} \sum_{i=1}^{n} a'_{1n} \\ \frac{1}{n} \sum_{i=2}^{n} a'_{2n} \\ \vdots \\ \frac{1}{n} \sum_{i=n}^{n} a'_{nn} \end{bmatrix}$$
(4)

A consistency index (C.I.) is thus obtained:

$$C.I. = \frac{\lambda_{\text{max}} - n}{n - 1} \tag{5}$$

where  $\lambda_{max}$  is the eigenvalue of the comparison matrix. The consistency judgment must be checked for an appropriate value of m by using the consistency ratio (C.R.) to ensure the consistency of the comparison matrix (Zou and Li 2008):



$$C.R. = \frac{C.I.}{R.I.} \tag{6}$$

where R.I. denotes the random index. The R.I. values for each value of m are listed in Supplemental Table 2 (Saaty 1980). If C.R. < 0.10, then weights can be used. If C.R.  $\ge 0.10$ , this denotes that the result is inconsistent judgment. If this is the case, then the comparison in the AHP needs to be revised (Chakraborty and Banik 2006; Demirel et al. 2012).

4. Establishment of FCI: there are many decision rules in GIS, of which the weighted linear combination model is used most often (Eastman 1999; Jiang and Eastman 2000). The weighted overlay, weighted linear average, and weighted summation also can be described as a weighted linear combination model (Malczewski 2006). The procedure of weighted linear combination is shown in Fig. 1. A local weighted linear combination that can be used in GIS can be developed based on the criteria range and weight (Carter and Rinner 2014), and based on the local weighted linear combination (Malczewski 2000, 2011), the FCI can be calculated:

$$FCI = \sum_{i=1}^{n} w'_{i} \cdot F_{i}(x, y) \tag{7}$$

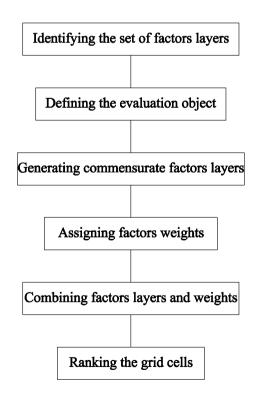


Fig. 1 The procedure of weighted linear combination

where  $w'_i = (w_1, w_2', ..., w_n)^T$  is the weight of the *i*th factor and  $F_i(x, y)$  is the value of the ith factor that affects the mine's FCI.

# **Case Study**

# **Geological Background**

The Chensilou coal mine is located in the city of Yongcheng in Henan province, China (Supplemental Fig. 2). According to geological drilling data, the lithology of the Chensilou coal mine consists of Quaternary (Q), Neogene (E), Permian (P), Carboniferous (C), and Ordovician (O) deposits. A total of 1008 faults have been exposed after excavating and exploitation. There are 21 faults with a throw of  $\geq$  30 m, 13 faults with a throw of > 30 m but  $\geq$  20 m, 65 faults with a throw of < 20 m but  $\geq$  10 m, and 909 faults with a throw of < 10 m. Rose diagrams were drawn to examine the characteristics of the faults with a throw of < 10 m (Supplemental Fig. 3). Of these faults, the fault structure of the E–W deposits is the primary feature characteristic, and that of the S–N deposits is the secondary feature characteristic.

The Chensilou coal mine faces the threat of water inrush from limestone aquifers of the Taiyuan Formation. Eleven limestone aquifers underlie an exploitable coal seam and can be recharged by each other. The degree of fault complexity and its relationship with water yield must be analyzed to prevent and control water inrush when mining takes place near these aquifers.

## **Evaluating Degree of Fault Complexity Based on FCI**

The dip angle of the coal seam, fault throw, and fractal dimensions of the faults are the three main factors that affect the degree of fault complexity in China's coal mines (State Administration of Coal Mine Safety 2014).

To calculate the fractal dimensions of the faults, the area in this study was divided into 42 boxes. The length of the side of each box was 500 m, as shown in Supplemental Fig. 4. The fractal dimension of each box can be obtained using GIS, as shown in Fig. 2, which shows the first box as an example. Then the map of the box with the faults is converted into raster format with different output grid cell sizes. The number of grid cells with different lengths is provided in Supplemental Table 3. Also, the fractal dimension of the first box is obtained from the double logarithmic image (Fig. 2c). Then, the fractal dimension of the other boxes is also calculated. The factors of the fractal dimensions of the faults were quantified and are listed in Supplemental Table 4.



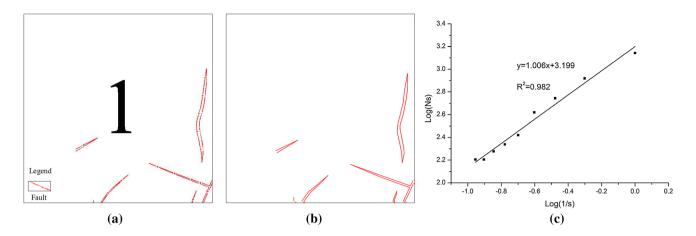


Fig. 2 Calculation process of fractal dimensions in GIS. a First box; b Raster format of first box; c double logarithmic image

## Results

Based on the GIS, a thematic map of the fractal dimensions was constructed using the natural breaks method (Jenks 1963) which is a means of data classification. Then, in order to eliminate the effects of dimensions between the factors, the thematic maps were normalized (Fig. 3a). According to the geological data from the excavation and exploration processes, the normalized thematic maps of the dip angle of the coal seam and fault throw are shown in Fig. 3b, c, respectively. The weight of each factor was calculated by using AHP. Thirty experts from academia and the coal mine were selected due to their expertise and consulted about the factors. The proportion of importance

of the fractal dimension, dip angle of the coal seam and fault throw were obtained as 35.7%, 31.5%, and 32.8% respectively. Then, the decision matrix was established as:

$$A = \begin{bmatrix} 1.000 & 2.000 & 3.000 \\ 0.500 & 1.000 & 1.200 \\ 0.333 & 0.833 & 1.000 \end{bmatrix}$$
 (8)

The weight of each factor was calculated and listed in Table 1. The eigenvalue of the comparison matrix,  $\lambda_{max} = 3$ , the C.I. = 0, and the C.R. = 0 < 0.10, can be used as weights. The thematic map of the FCI is shown in Fig. 4a. The FCI is less than 0.600 for 90% of the studied area.

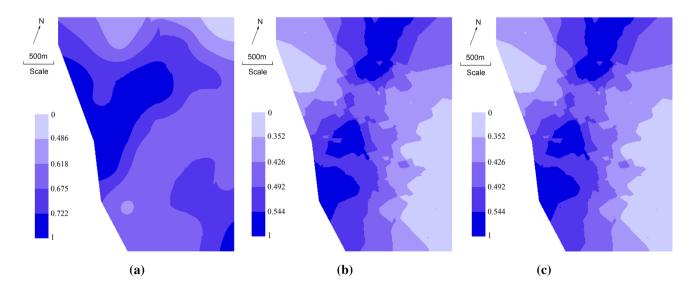


Fig. 3 Normalized thematic maps. a Fractal dimension; b dip angle of coal seam; c fault throw



Table 1 Weight of factors

Evaluation object	Factors	Weight	C.R.	$\lambda_{\text{max}}$
The degree of fault complexity	Fractal dimension	0.55	0.00	3.00
	Dip angle of the coal seam	0.25		
	Fault throw	0.20		

Fig. 4 Thematic map of FCI. a The FCI based on AHP; b the FCI based on comprehensive weights

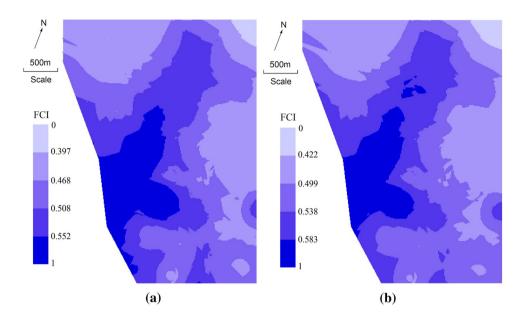


Table 2 Entropy and weight of each factor

Factors	Fractal dimension	Dip angle of the coal seam	Fault throw
Entropy	0.9216	0.9668	0.9764
Weight	0.5799	0.2454	0.1747

# Discussion

There are two methods to determine the weight: one is subjective weighting, which is based on the experience of experts, the other is objective weighting, which is based on criterion or factor values. The weighting process for the three factors that influence the degree of fault complexity is only subjective according to AHP model. Hence, an information entropy model was used as an objective weighting method to improve the factor weight information. Entropy can be used to measure the factor information. The weight of factors can be obtained by calculating the factor information. The information entropy of the factor can be defined as:

$$\begin{cases} E_{j} = -\frac{\sum_{i=1}^{m} p_{ij} \ln(p_{ij})}{\ln(m)} \\ p_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \end{cases}$$
(9)

where  $E_j$  is the entropy of the *j*th factor, m is the number of the *j*th factor,  $a_{ij}$  is the value of the *j*th factor, and  $p_{ij}$  is the rate of the *j*th factor. The weight of the *j*th factor is:

$$w_{E_j} = \frac{1 - E_j}{\sum_{j=1}^{n} (1 - E_j)}$$
 (10)

Based on Fig. 3a–c, the entropy of each factor was obtained; the weight of each factor is listed in Table 2. The factor weights calculated by the AHP model differ from those from those from the information entropy model. The average values of the factor weights were calculated so that both the subjective and objective information could be considered. The resulting evaluation factor weight vectors were 0.5650, 0.2477, and 0.1873. Based on the comprehensive weights, the improved FCI thematic map is shown in Fig. 4b. In the southeast of the study area, the FCI in Fig. 4b is more accurate than in Fig. 4a.

The location and its water yield have been exposed in the studied area during excavation. Figure 5 shows the variation in water yield vs the FCI; the quantitative relationship between water yield and FCI is defined as:

$$y = 1.034 \times 10^{-4} \exp(11.442x) - 0.002; R = 0.995$$
 (11) where y is the water yield, and x is the FCI.

Moreover, the water yield in the studied area increases with a higher FCI, and when the FCI=0.569, water inrush



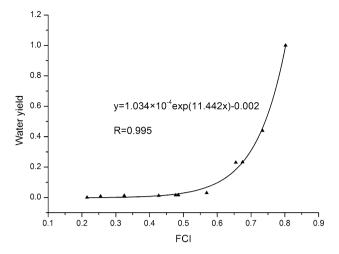


Fig. 5 Water yield versus FCI

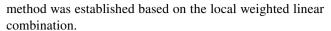
Table 3 Degree of fault complexity in studied area

Type of fault	Faults complexity index (FCI)	Proportion (%)
Very simple	FCI≤0.538	71.1
Simple	$0.508 < FCI \le 0.569$	10.5
Moderate	$0.569 < F CI \le 0.734$	13.4
Complex	$0.734 < FCI \le 0.812$	4.5
Very complex	FCI>0.812	0.5

takes place, with a water yield of 35 m<sup>3</sup>/h. When the FCI is 0.734 and 0.802, the yield of the inrush is 383 and 860 m<sup>3</sup>/h, respectively. When the FCI is 0.812, the water yield can reach or even exceed 1000 m<sup>3</sup>/h. Therefore, FCI values of 0.569, 0.73, and 0.812 were selected as fault complexity thresholds in this coal mine. The degree of fault complexity was then divided into five levels of complexity in the studied area (listed in Table 3 and shown in Supplemental Fig. 5). Also, 95% of the faults in the studied area are very simple, simple, and moderate faults. The fault complexity in the studied area therefore consists of simple and moderate faults.

## **Conclusions**

This paper presents a new method for evaluating the degree of fault complexity in coal mines based on GIS and the fractal theory. The three main factors that affect the degree of complexity of coal mine faults are: the dip angle of the coal seam, fault throw, and fractal dimensions of the faults. A FCI was proposed to evaluate the degree of fault complexity in the mines. An AHP model was used to calculate the factor weights. The fractal theory is used to determine the fractal dimensions of the studied area. The FCI calculation



The method was validated through a case study of the Chensilou coal mine in Henan Province, China. An information entropy model was used as an objective weighting method to improve the factor weight information. The average factor weight of the values was calculated so that both the subjective and objective information could be contained in the evaluation result. Also, the relationship between the water yield and FCI was examined to obtain the thresholds for the degree of fault complexity of the studied area. The degree of fault complexity of the studied area was quantified and divided into five levels of complexity. The studied area mostly has simple and moderate faults, and the water yield in the Chensilou coal mine increases with a higher FCI. More water yield data can be exposed during excavation, so the model can be improved in the future.

**Acknowledgements** The authors acknowledge financial support from the National Key R&D Program of China under Grant 2017YFC0804101. We also thank the Yongcheng Coal and Electricity Holding Group Co. Ltd for providing data support.

## References

Adib A, Afzal P, Ilani SM, Aliyari F (2017) Determination of the relationship between major fault and zinc mineralization using fractal modeling in the Behabad fault zone, central Iran. J Afr Earth Sci 134:308–319

Aviles CA, Scholz CH, Boatwright J (1987) Fractal analysis applied to characteristic segments of the San Andreas fault. J Geophys Res Sol Ea 92(B1):331–344

Babanouri N, Nasab SK, Sarafrazi S (2013) A hybrid particle swarm optimization and multi-layer perceptron algorithm for bivariate fractal analysis of rock fractures roughness. Int J Rock Mech Min Sci 60:66–74

Ben-Zion Y, Sammis CG (2003) Characterization of fault zones. In: Seismic Motion, Lithospheric Structures, Earthquake and Volcanic Sources: the Keiiti Aki Volume. Birkhäuser, Basel, pp 677–715

Carpinteri A, Paggi M (2008) Size scale effects on strength, friction and fracture energy of faults: a unified interpretation according to fractal geometry. Rock Mech Rock Eng 41(5):735–746

Carpinteri A, Paggi M (2009) A fractal interpretation of size scale effects on strength, friction and fracture energy of faults. Chaos Soliton Fract 39(2):540–546

Carter B, Rinner C (2014) Locally weighted linear combination in a vector geographic information system. J Geograph Syst 16(3):343–361

Chakraborty S, Banik D (2006) Design of a material handling equipment selection model using analytic hierarchy process. Int J Adv Manuf Tech 28(11):1237–1245

Chen JF, Zhu ZJ, Yan CD (1999) Fractal dimension description of fault density in a coal mine. Coal Geol Chin (3):8–10 (In Chinese)

Demirel N, Yücenur GN, Demirel T, Muşdal H (2012) Risk-based evaluation of Turkish agricultural strategies using fuzzy AHP and fuzzy ANP. Human Ecolog Risk Assess Int J 18(3):685–702

Duan LH, Ran DL (2013) Nonlinear analysis of coal mine water inflows and fault structure. Chin Coal 9:32–35 (In Chinese)



- Eastman JR (1999) Multi criteria evaluation and GIS. Geogr Inf Syst 1(1):493–502
- Guarnieri P (2002) Regional strain derived from fractal analysis applied to strike slip fault systems in NW Sicily. Chaos Soliton Fract 14(1):71–76
- Han J, Shi LQ, Yu XG, Wei JC, Li SC (2009) Mechanism of mine water inrush through a fault from the floor. Int J Min Sci Tech 19(3):276–281
- Hirata T (1989) Fractal dimension of fault systems in Japan: fractal structure in rock fracture geometry at various scales. Pure Appl Geophys 131(1):157–170
- Hossain MS, Kruhl JH (2015) Fractal geometry based quantification of shock-induced rock fragmentation in and around an impact crater. Pure Appl Geophys 172(7):2009–2023
- Jenks GF (1963) Generalization in statistical mapping. Ann Assoc Am Geogr 53(1):15–26
- Jiang H, Eastman JR (2000) Application of fuzzy measures in multicriteria evaluation in GIS. Int J Geogr Inf Sci 14(2):173–184
- Kato N, Lei X (2001) Interaction of parallel strike slip faults and a characteristic distance in the spatial distribution of active faults. Geophys J Int 144(1):157–164
- Li J, Feng Q, Guo QL (2008) Fractal study of sustainable proportions of natural and artificial oases. Environ Geol 55(7):1389–1396
- Li LC, Yang TH, Liang ZZ, Zhu WC, Tang CN (2011) Numerical investigation of groundwater outbursts near faults in underground coal mines. Int J Coal Geol 85(3–4):276–288
- Makropoulos CK, Butler D (2006) Spatial ordered weighted averaging: incorporating spatially variable attitude towards risk in spatial multi-criteria decision-making. Environ Modell Softw 21(1):69–84
- Malczewski J (2000) On the use of weighted linear combination method in GIS: common and best practice approaches. Trans GIS 4(1):5–22
- Malczewski J (2006) GIS based multicriteria decision analysis: a survey of the literature. Int J Geogr Info Sci 20(7):703–726
- Malczewski J (2011) Local weighted linear combination. Trans GIS 15(4):439–455
- Mandelbrot BB (1979) Fractals: form, chance and dimension. WH Freeman and Co., San Francisco
- Mandelbrot BB (1983) The fractal geometry of nature. W. H. Freeman, New York City
- Miao XX, Liu WQ, Chen ZQ (2004) Dynamics of systems of seepage flow in surrounding rock affected by mining. Science Press, Beijing (In Chinese)
- Network of coal mine safety production in China (2018) Accident management. http://www.mkaq.org/sggl/. Accessed 7 May 2018
- Öncel AO, Main I, Alptekin Ö, Cowie P (1996) Spatial variations of the fractal properties of seismicity in the Anatolian fault zones. Tectonophysics 257(2–4):189–202

- Pérez López R, Paredes C, Muñoz Martín A (2005) Relationship between the fractal dimension anisotropy of the spatial faults distribution and the paleostress fields on a Variscan granitic massif (Central Spain): the F parameter. J Struct Geol 27(4):663–677
- Saaty TL (1980) The analytic hierarchy process. McGraw Hill, New York City
- San Cristóbal JR (2011) Multi criteria decision making in the selection of a renewable energy project in Spain: the Vikor method. Renew Energ 36(2):498–502
- Sengupta P, Nath SK, Thingbaijam KKS, Mistri S (2011) Fractal analysis of major faults in India on a regional scale. J Geol Soc India 78(3):226
- State Administration of Coal Mine Safety (2014) Regulations of geological work for coalmines. Coal Industry Press, Beijing (In Chinese)
- Sukmono S, Zen MT, Kadir WGA, Hendrajaya L, Santoso D, Dubois J (1996) Fractal geometry of the Sumatra active fault system and its geodynamical implications. J Geodyn 22(1–2):1–9
- Uyan M (2013) GIS-based solar farms site selection using analytic hierarchy process (AHP) in Karapinar region, Konya or Turkey. Renew Sust Energ Rev 28:11–17
- Wang Y, Yang WF, Li M, Liu X (2012) Risk assessment of floor water inrush in coal mines based on secondary fuzzy comprehensive evaluation. Int J Rock Mech Min Sci 52(6):50–55
- Wang X, Wang T, Wang Q, Liu X, Li R, Liu B (2017) Evaluation of floor water inrush based on fractal theory and an improved analytic hierarchy process. Mine Water Environ 36(1):87–95
- Wu Q, Zhou WF (2008) Prediction of groundwater inrush into coal mines from aquifers underlying the coal seams in China: vulnerability index method and its construction. Environ Geol 56(2):245–254
- Wu Q, Wang M, Wu X (2004) Investigations of groundwater bursting into coal mine seam floors from fault zones. Int J Rock Mech Min Sci 41(4):557–571
- Xu D (2011) Advances in the research on mechanisms of the groundwater inrush caused by the fault reactivation in coalmines. Proc Eng 26:824–831
- Yang BB, Sui WH, Duan LH (2017) Risk assessment of water inrush in an underground coal mine based on GIS and fuzzy set theory. Mine Water Environ 36(4):617–627
- Zhao J, Chen S, Zuo R, Carranza EJM (2011) Mapping complexity of spatial distribution of faults using fractal and multifractal models: vectoring towards exploration targets. Comput Geosci 37(12):1958–1966
- Zou X, Li D (2008) A multidisciplinary GIS-based approach for the potential evaluation of land consolidation projects: a model and its application. In: Proceedings of WSEAS International Conf on Mathematics and Computers in Science and Engineering (No. 7), World Scientific and Engineering Academy and Soc

